New results from Singapore on non-Ramsey sets

Definition $\omega = \{0, 1, 2, \ldots\}$.

Definition For $A \subseteq \omega$, $[A]^\omega =_{\mathrm{df}} \{B \mid B \subseteq A \;\&\; B \text{ infinite}\}$.

Definition For A and B in $[\omega]^\omega$, write $A \sim B$ if the symmetric difference $A \triangle B$ is finite.

Definition $\pi : [\omega]^\omega \longrightarrow \{0, 1\}$ is *invariant* if $A \sim B \implies \pi(A) = \pi(B)$.

Definition π is a *chameleon* if $n \in A \in [\omega]^\omega \implies \pi(A \setminus \{n\}) \neq \pi(A)$.

Thus chameleons are far from invariant.

Remark Chameleons have been studied by three members of ERMIT in collaboration with Carlos Di Prisco of Caracas.

Definition The partition relation $\omega \longrightarrow (\omega)^\omega$ means that for each $\pi : [\omega]^\omega \longrightarrow \{0, 1\}$, there is an infinite A with π constant on $[A]^\omega$.

Such π are called *Ramsey.* The existence of non-Ramsey π follows from AC: I showed in 1968 that it is consistent with DC, the Axiom of Dependent Choice, which is known to be strictly weaker than AC, that all π are Ramsey. My proof assumed the consistency of the existence of a strongly inaccessible cardinal; whether that assumption is necessary is still unknown.

Remark There is a link with mathematical economics: Luc Lauwers of Louvain showed in 2006 that the existence of non-Ramsey π follows from the existence of a Paretian and finitely anonymous ordering of the set of infinite utility streams.

Definition AD is the Axiom of Determinacy, which says of certain games of infinite length between two players that one of the players has a winning strategy.

It was shown fifty years ago that $\mathsf{AD}+\ \mathsf{DC}$ proves that every set of reals is Lebesgue measurable and has the property of Baire. The following has remained open:

Problem Does $\mathsf{AD}+\ \mathsf{DC}$ prove that all π are Ramsey ?

The recently completed Ph. D. thesis by Dongxu Shao in Singapore makes significant progress towards this problem by proving, assuming $\mathsf{AD}+\ \mathsf{DC}$, that if every invariant π is Ramsey then every π is Ramsey.

In this talk I shall aim to present Shao's proof.

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