

1)

$$\frac{q \quad \frac{p \quad p \rightarrow (q \rightarrow r)}{(q \rightarrow r)}}{r}$$

2)

$$\frac{\frac{[p \wedge \neg q]}{p} \quad p \rightarrow q \quad [p \wedge \neg q]}{q \quad \neg q} \quad \perp}{\neg(p \wedge \neg q)}$$

3)

$$\frac{[p] \quad p \rightarrow q}{q \quad \neg q} \quad \perp}{\neg p}$$

4)

$$\frac{\frac{[p]}{p \vee \neg p} \quad \perp}{\neg p} \quad \frac{[\neg p]}{p \vee \neg p} \quad \perp}{p} \quad \perp}{p \vee \neg p}$$

5)

$$\frac{(\neg p \vee \neg q) \quad \frac{[p \wedge q]}{p} \quad [\neg p] \quad \perp \quad \frac{[p \wedge q]}{q} \quad [\neg q] \quad \perp}{\perp}}{\neg(p \wedge q)}$$

6)

$$\frac{\begin{array}{c} \triangle \\ \dots * \\ p \vee \neg p \end{array} \quad \frac{[p] \quad p \rightarrow q}{q} \quad \frac{[\neg p]}{[\neg p]}}{\neg p \vee q} \quad \frac{[\neg p]}{[\neg p]}}{\neg p \vee q}}{\neg p \vee q}$$

7)

$$\frac{p \vee q \quad \frac{[p] \quad [\neg p]}{\perp}}{q} \quad [q]}{\neg p \rightarrow q}$$

8)

$$\frac{[p] \quad p \rightarrow q \quad [q] \quad \frac{[\neg p] \quad \neg p \rightarrow \neg q}{\neg q} \quad \perp}{q} \quad \frac{[\neg p]}{p}}{p \leftrightarrow q}$$