The Curry-Howard correspondence:

- · propositions as types
- proofs as programs
- simplification of proofs as evaluation of programs

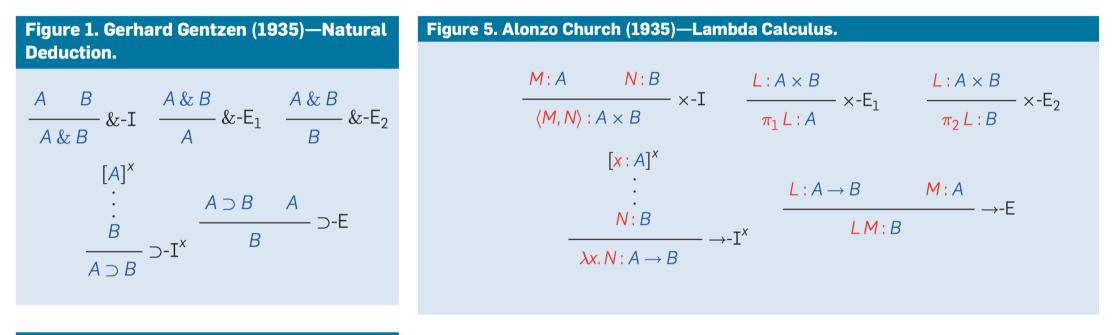


Figure 2. A proof.

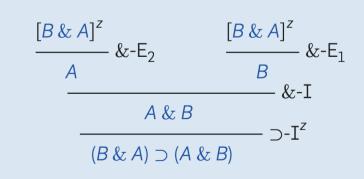


Figure 6. A program.

$$\frac{\begin{bmatrix} z:B \times A \end{bmatrix}^{z}}{\frac{\pi_{2} z:A}{} \times -E_{2}} \qquad \frac{\begin{bmatrix} z:B \times A \end{bmatrix}^{z}}{\pi_{1} z:B} \times -E_{1}$$

$$\frac{\langle \pi_{2} z, \pi_{1} z \rangle:A \times B}{\langle \pi_{2} z, \pi_{1} z \rangle:A \times B} \rightarrow -I^{z}$$

$$\frac{\lambda z \cdot \langle \pi_{2} z, \pi_{1} z \rangle:(B \times A) \rightarrow (A \times B)}{\langle \pi_{2} z, \pi_{1} z \rangle:(B \times A) \rightarrow (A \times B)}$$

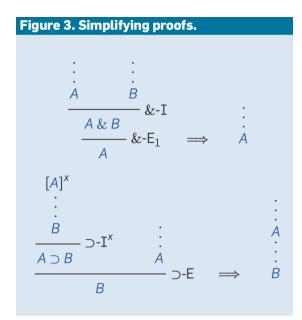


Figure 7. Evaluating programs.

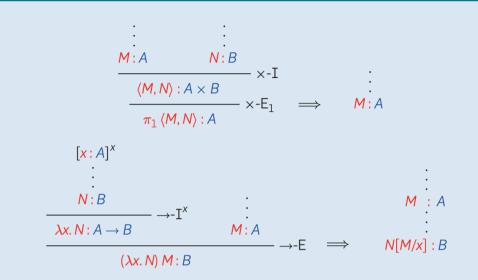


Figure 4. Simplifying a proof.

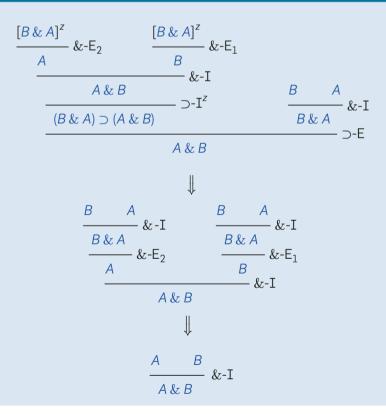


Figure 8. Evaluating a program. $\frac{\frac{[z:B \times A]^{z}}{\pi_{2} z:A} \times E_{2}}{\langle \pi_{1} z:B \rangle \times E_{1}} \xrightarrow{\pi_{1} z:B} \times E_{1}}{\langle \pi_{2} z, \pi_{1} z\rangle : A \times B} \longrightarrow I^{z} \xrightarrow{y:B} \xrightarrow{x:A} \times I}{\langle y, x\rangle : B \times A} \rightarrow E$ $(\lambda z. \langle \pi_2 z, \pi_1 z \rangle) \langle y, x \rangle : A \times B$ $\begin{array}{c} \downarrow \\ \frac{y:B \quad x:A}{\langle y,x\rangle:B \times A} \times -I \\ \frac{\pi_2 \langle y,x\rangle:A}{\langle y,x\rangle:A} \times -E_2 \end{array} \quad \begin{array}{c} \frac{y:B \quad x:A}{\langle y,x\rangle:B \times A} \times -I \\ \frac{\langle y,x\rangle:B \times A}{\pi_1 \langle y,x\rangle:B} \times -I \end{array}$ $\frac{1}{\langle \pi_2 \langle y, x \rangle, \pi_1 \langle y, x \rangle \rangle : A \times B} \times -\mathbf{I}$ $\frac{\mathbf{x}:A \qquad \mathbf{y}:B}{------} \times \mathbf{I}$ $\langle x, y \rangle : A \times B$